

LA-UR-19-26644

Approved for public release; distribution is unlimited.

Title: Ray Trace Modeling Code

Author(s): Light, Max Eugene

Intended for: Report

Issued: 2019-07-15

Disclaimer:

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by Triad National Security, LLC for the National Nuclear Security Administration of U.S. Department of Energy under contract 89233218CNA000001. By approving this article, the publisher recognizes that the U.S. Government retains nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

Ray Trace Modeling Code

Max Light

July 12, 2019

Abstract

This document gives a brief overview of the basic theory of electromagnetic (EM) wave propagation through an inhomogeneous, non-stochastic plasma and the development of a computer code to model that propagation in three dimensional space. This code has been developed to account for extreme ionospheric refractory conditions, that is, propagation of electromagnetic waves near the geomagnetic poles, and significant electron density gradients in all three directions.

Contents

1	Introduction	2
2	Theory	3
3	Computer Algorithm	6
3.1	Overview	6
3.2	Algorithm	7
4	Application	10
5	Summary	13

Introduction

Accurate characterization of radio frequency (RF) signals that have traversed the ionosphere is very important for developing and characterizing detection systems. This characterization is based on the theory of ionospheric EM wave propagation. For this report, only the non-stochastic part of the ionospheric plasma will be addressed in terms of EM propagation, that is, the ionospheric plasma is assumed to be quiescent. EM scintillation due to the stochastic part of the ionosphere is addressed elsewhere.

We have several codes to address ionospheric refraction in mostly nominal refractory conditions, but the nature of extreme ionospheric refractory conditions requires a fully three dimensional treatment. This is due to the large changes in the geomagnetic field and electron density responsible for the extreme refraction.

Theory

The ionosphere is basically a non homogeneous magnetized plasma, and there are a number of different ways to model EM wave propagation through this type of medium. The basics of EM wave propagation in an inhomogeneous plasma are well documented [1, 2, 3, 4]. We wish to solve the RF signal's (EM wave's) amplitude and phase at a given frequency as it traverses the ionosphere. The EM wave frequency and plasma parameter regime of this work are best suited to the well known ray tracing technique [5, 6, 1, 2, 7] to find this solution.

Ray tracing is based on the assumption that the wave fields can be expanded into local *plane* waves, which is an excellent assumption considering the distances involved with a signal originating on the Earth's surface and propagating up to the underside of the ionosphere. The technique renders a solution for the spatial trajectory and amplitude of a 'ray' representing the EM wave's Poynting vector. A single component of the wave's electric field E can thus be represented as an asymptotic expansion in powers of $1/k_0$ [7]

$$E(\mathbf{r}) = e^{ik_0\Psi(\mathbf{r},t)} \sum_{m=0}^{\infty} \frac{E_m(\mathbf{r})}{(ik_0)^m} \quad (2.1)$$

where $k_0 = \omega/c$ is the wave vector number, ω is the radian frequency, and c is the speed of light, all in *vacuum*. The time dependence of E is harmonic, that is $e^{i\omega t}$, and will be assumed throughout.

Inside the medium, the local wave vector and frequency are defined as

$$\mathbf{k}(\mathbf{r}, t) = \nabla\Psi \quad \omega(\mathbf{r}, t) = -\frac{\partial\Psi}{\partial t} \quad (2.2)$$

The wave (Helmholtz) equation for EM waves is

$$\nabla^2 E(\mathbf{r}) + k_0^2 n(\mathbf{r})^2 E = 0 \quad (2.3)$$

where $n(\mathbf{r})$ is the refractive index of the medium. Substitute 2.1 into 2.3 and equate like powers of k_0

$$(\nabla\Psi)^2 = n^2 \quad (2.4)$$

$$2\nabla\Psi \cdot \nabla E_0 + E_0 \nabla^2\Psi = 0 \quad (2.5)$$

\vdots

$$2\nabla\Psi \cdot \nabla E_m + E_m \nabla^2\Psi = -\nabla^2 E_{m-1} \quad (2.6)$$

Equation 2.4 is known as the *eikonal* equation and is solved such that the ray's trajectory is constrained to be along a path such that the dispersion relation of the EM wave is satisfied at

every point of the path. Equations 2.5 and 2.6 describe the *amplitude transport* of the wave along the ray path.

The first term in the expansion of E in 2.1 will dominate as long as the spatial variation of the index of refraction is small compared to the wavelength, or

$$\frac{|\nabla n(\mathbf{r})|}{k_0^2 n^2(\mathbf{r})} \ll 1 \quad (2.7)$$

This assumption is known as the geometric optics limit of the ray tracing solution [7], and applies for the parameters of the ionospheric plasma and wavelengths related to this work.

Taking the first term in the wave field expansion from 2.1

$$E(\mathbf{r}) = E_0 e^{ik_0 \Psi(\mathbf{r}, t)} \quad (2.8)$$

and using it in 2.3, we arrive at an equation of the form

$$\mathfrak{L} \left(\nabla, \frac{\partial}{\partial t}, \mathbf{r}, t \right) \cdot E_0(\mathbf{r}) = 0 \quad (2.9)$$

where the operator \mathfrak{L} incorporates the dispersion relation of the EM wave propagating at any point in the medium $D(\omega, k, r)$ as

$$\text{Det}[\mathfrak{L}] = D(\omega, \mathbf{k}, \mathbf{r}) = 0 \quad (2.10)$$

and

$$D(\omega, \mathbf{k}, \mathbf{r}) = 1 - \frac{X(1 - X)}{1 - X - \frac{1}{2}Y^2 \sin^2 \theta \pm \left[\frac{1}{4}Y^4 \sin^4 \theta + Y^2 \cos^2 \theta (1 - X)^2 \right]^{1/2}} - \frac{c^2 k^2}{\omega^2} \quad (2.11)$$

where $D(\omega, \mathbf{k}, \mathbf{r})$ is the well known Appleton-Hartree dispersion relation for a EM wave propagating in a magnetized plasma [1, 3]. In 2.11, θ is the angle between the local magnetic field B_0 and the wave number $\mathbf{k} = \nabla \Psi$. $X(\omega, \mathbf{r})$ and $Y(\omega, \mathbf{r})$ are defined as $\omega_{pe}(\mathbf{r})^2/\omega^2$ and $\omega/\omega_c(\mathbf{r})$ respectively where

$$\omega_{pe}(\mathbf{r})^2 = \frac{n_e(\mathbf{r})e^2}{\epsilon_0 m_e}; \quad \omega_c = \frac{eB_0(\mathbf{r})}{m_e} \quad (2.12)$$

and n_e , m_e , B_0 , and ϵ_0 are the local electron density, electron mass, local magnetic field magnitude, and free space permeability respectively.

Solving the partial differential system of equations in 2.9 by the method of characteristics, gives a set known as the ray equations

$$\frac{dR}{dt} = -\frac{\partial D}{\partial \mathbf{k}} / \frac{\partial D}{\partial \omega} \quad (2.13)$$

$$\frac{d\mathbf{k}}{dt} = -\frac{\partial D}{\partial \mathbf{R}} / \frac{\partial D}{\partial \omega} \quad (2.14)$$

where R is the position vector along the ray path and t is the group time. The right side of equation 2.13 is effectively the group velocity, and thus the change in R is due to the change in the group velocity. Equation 2.13 shows that changes in \mathbf{k} are due to the spatial gradients in the plasma from the dispersion relation.

In general, the solutions to the dispersion relation 2.11 are complex if damping such as electron collisions are included, however, these effects are not included here, and thus only the real parts of the solutions are necessary.

We can now solve the trajectory of the ray from which we can get the time (phase) delay required to propagate through the medium at a given frequency, as well as the path of the ray.

To solve the amplitude along the ray path, the amplitude transport equation 2.5 can be used. Alternatively, we can track the cross section of ‘bundles of rays’ [7], or divide the initial amplitude of the ray by the total path length of the ray to get the amplitude at the end of the ray. The last two methods are adequate approximations as long as refractive effects are not extremely strong.

Note that the solution of the wave’s path and amplitude are for a single frequency. wide bandwidth signal propagation is found by using many discrete frequencies in the signal bandwidth, solving their amplitudes and phases, and constructing a frequency domain transfer function.

Equation 2.11 is bi-quadratic and has two principle roots [1]. This comes from the \pm sign in the denominator of the second term on the right side. Thus, for a given frequency, one or both modes will propagate, and this will affect the amplitude of the ray after it leaves the ionosphere. The proportion of each mode inside the medium, related to the initial amplitude of the ray before entering it, is a separate issue currently being pursued, and will not be addressed in this work.

Computer Algorithm

It is common to solve the eikonal equation for the trajectory of the ray, and find the amplitude at the end of the ray by dividing the initial amplitude by the ray's path length. This avoids lengthy calculation of the ray's amplitude from 2.5 and 2.6, but as mentioned earlier, can lead to incorrect amplitudes for extreme refraction. In this work, we solve for the ray paths and find the magnitudes and phases of the amplitudes by a division of the total path length at each frequency (and mode), since extreme refraction would lead to a ray missing the receiver altogether or being refracted back towards the transmitter. In either case, the ray will not contribute to the *received* signal.

3.1 Overview

We use the method of Horne [8] to solve for the ray trajectories. The approach is to solve equations 2.13 and 2.14 by defining two coordinate systems as shown in Figure 3.1. The first coordinate system is the Earth Centered Earth Fixed (ECEF) coordinate system defined as $(0XYZ)$. A point P on the ray path makes an angle θ with the Z axis and ϕ with the X axis. The second system is a local coordinate system at that (or any) point along the ray path defined as $(Pxyz)$. In this system, the z axis is parallel to the local magnetic field B_0 , the x axis is orthogonal to z and lies in the meridian plane pointing away from Earth at the equator, and the y axis completes the right-handed system [8]. The wave vector makes an angles Ψ with the z axis and η with the x axis.

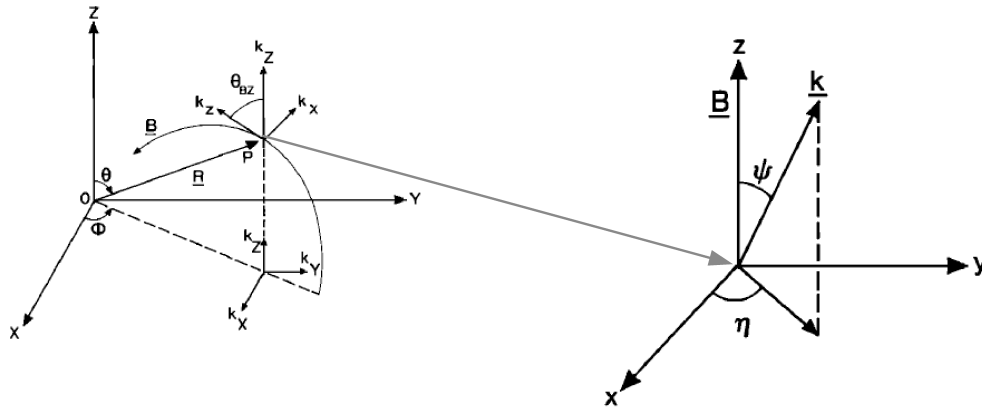


Figure 3.1: Geometry for solving the ray equations 2.13, 2.14 from [8].

The dispersion relation $D(\omega, \mathbf{k}, \mathbf{r})$ is calculated in the local system, with

$$k_{\perp}^2 = k_x^2 + k_y^2 \quad (3.1)$$

Equations 2.13 and 2.14 are then solved in the ECEF system by using coordinate transforms.

The coordinate transform for the wave vector using the rotation matrix from local to ECEF is then

$$\begin{bmatrix} k_X \\ k_Y \\ k_Z \end{bmatrix} = \begin{bmatrix} \cos \theta_{BZ} \cos \phi & -\sin \phi & -\sin \theta_{BZ} \cos \phi \\ \cos \theta_{BZ} \sin \phi & \cos \phi & -\sin \theta_{BZ} \sin \phi \\ \sin \theta_{BZ} & 0 & \cos \theta_{BZ} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad (3.2)$$

Now, the numerator on the right hand side of 2.13 is

$$\begin{bmatrix} \frac{\partial D}{\partial k_X} \\ \frac{\partial D}{\partial k_Y} \\ \frac{\partial D}{\partial k_Z} \end{bmatrix} = \begin{bmatrix} k_x \cos \theta_{BZ} \cos \phi - k_y \sin \phi & -\sin \theta_{BZ} \cos \phi \\ k_x \cos \theta_{BZ} \sin \phi - k_y \cos \phi & -\sin \theta_{BZ} \sin \phi \\ k_x \sin \theta_{BZ} & \cos \theta_{BZ} \end{bmatrix} \begin{bmatrix} \frac{1}{k_{\perp}} \frac{\partial D}{\partial k_{\perp}} \\ \frac{\partial D}{\partial k_z} \end{bmatrix} \quad (3.3)$$

By the chain rule, the numerator on the right hand side of equation 2.14 is

$$\frac{\partial D}{\partial \mathbf{R}} = \frac{\partial D}{\partial \mathbf{B}} \frac{\partial \mathbf{B}}{\partial \mathbf{R}} + \frac{\partial D}{\partial n_e} \frac{\partial n_e}{\partial \mathbf{R}} + \frac{\partial D}{\partial \mathbf{k}} \frac{\partial \mathbf{k}}{\partial \mathbf{R}} \quad (3.4)$$

The first two differentials are straightforwardly solved once the magnetic field and electron density spatial profiles are known. The last term is calculated in the local system and transformed to the ECEF system.

$$\left[\frac{\partial D}{\partial \mathbf{k}} \frac{\partial \mathbf{k}}{\partial \mathbf{R}} \right]_X = T_1 \frac{\partial \theta}{\partial X} + T_2 \frac{\partial \phi}{\partial X} \quad (3.5)$$

$$\left[\frac{\partial D}{\partial \mathbf{k}} \frac{\partial \mathbf{k}}{\partial \mathbf{R}} \right]_Y = T_1 \frac{\partial \theta}{\partial Y} + T_2 \frac{\partial \phi}{\partial Y} \quad (3.6)$$

$$\left[\frac{\partial D}{\partial \mathbf{k}} \frac{\partial \mathbf{k}}{\partial \mathbf{R}} \right]_Z = T_1 \frac{\partial \theta}{\partial Z} + T_2 \frac{\partial \phi}{\partial Z} \quad (3.7)$$

where

$$T_1 = \frac{k_z k_x}{k_{\perp}} \frac{\partial D}{\partial k_{\perp}} - k_x \frac{\partial D}{\partial k_z} \quad (3.8)$$

$$T_2 = k_y \sin \theta_{BZ} \left(\frac{k_z}{k_{\perp}} \frac{\partial D}{\partial k_{\perp}} - \frac{\partial D}{\partial k_z} \right) \quad (3.9)$$

The derivatives of D can be calculated in a straightforward manner.

3.2 Algorithm

A computer ray tracing code has been implemented that can calculate the ray trajectory, amplitude, and time of flight (phase) for EM propagation through the ionosphere. The code can accept density and magnetic field profiles in functional form or in tabular form in all three dimensions. The calculation of the ray trajectory from equations 2.13 and 2.14 proceeds as follows:

0 Start at the Earth's surface where $n_e \rightarrow 0$ and $[(k_X^2 + k_Y^2 + k_Z^2)^{1q/2} = k_0$.

1 Specify the mode (+ or - in denominator of second term in 2.11), frequency, position on Earth's surface (X, Y, Z), the magnetic field B_0 , and θ_{BZ} where

$$\theta_{BZ} = \cos^{-1} \left[\frac{\mathbf{B}_0 \cdot \hat{Z}}{|\mathbf{B}_0| |\hat{Z}|} \right] \quad (3.10)$$

2 Perform necessary coordinate transforms via 3.2.

3 calculate

$$\frac{\partial D}{\partial k_{\perp}}, \frac{\partial D}{\partial k_z}, \frac{\partial D}{\partial \omega}, \frac{\partial D}{\partial \mathbf{B}}, \frac{\partial D}{\partial n_e}, \frac{\partial \mathbf{B}}{\partial \mathbf{R}}, \frac{\partial n_e}{\partial \mathbf{R}}$$

4 calculate

$$\frac{\partial D}{\partial \mathbf{k}}$$

in the ECEF coordinate system via

$$\frac{\partial D}{\partial k_{\perp}}, \frac{\partial D}{\partial k_z}$$

and equation 3.3

5 for a time step dt :

$$\Delta \mathbf{R} = \Gamma \times dt$$

where

$$\Gamma = -\frac{\partial D}{\partial \mathbf{k}} / \frac{\partial D}{\partial \omega}$$

and then

$$\mathbf{R}_{new} = \mathbf{R}_{old} + \Delta \mathbf{R}$$

6 calculate T_1 and T_2 from 3.8 and 3.9, and use these and \mathbf{R}_{new} to calculate

$$\frac{\partial D}{\partial \mathbf{k}} \frac{\partial \mathbf{k}}{\partial \mathbf{R}}$$

7 for the same time step dt , in the ECEF coordinate system

$$\Delta \mathbf{k} = \gamma \times dt$$

where

$$\gamma = \frac{\frac{\partial D}{\partial \mathbf{B}} \frac{\partial \mathbf{B}}{\partial \mathbf{R}} + \frac{\partial D}{\partial n_e} \frac{\partial n_e}{\partial \mathbf{R}} + \frac{\partial D}{\partial \mathbf{k}} \frac{\partial \mathbf{k}}{\partial \mathbf{R}}}{\partial D / \partial \omega}$$

from equation 3.4, and thus

$$\mathbf{k}_{new} = \mathbf{k}_{old} + \Delta \mathbf{k}$$

8 Go to **2** for the next iteration.

This process is repeated until the desired limit in time or ray length is reached. For each frequency/mode, the amplitude can be found from dividing the initial amplitude by the total ray path length, and the phase found from the time taken to travel the path length.

Outside of the plasma, the dispersion relation for the EM wave is much simpler

$$D(\omega, \mathbf{k}, \mathbf{r}) = 1 - \frac{c^2 |\mathbf{k}|^2}{\omega^2} = 0 \quad (3.11)$$

and it is also much simpler to trace the ray, since it will only travel in a straight trajectory in vacuum. Therefore, the code must check to verify that the electron density is sufficiently close to zero in order to transfer to the vacuum dispersion relation. This is done with the help of an alternate form of the Appleton-Hartree dispersion relation [1]

$$An^4 - Bn^2 + C = 0 \quad (3.12)$$

where the index of refraction is n and

$$A = S \sin^2 \theta + P \cos^2 \theta \quad (3.13)$$

$$B = RL \sin^2 \theta + PS(1 + \cos^2 \theta) \quad (3.14)$$

$$C = PRL \quad (3.15)$$

$$R = 1 - \frac{X}{1 - Y^2} (1 - Y) \quad (3.16)$$

$$L = 1 - \frac{X}{1 - Y^2} (1 + Y) \quad (3.17)$$

$$S = 1 - \frac{X}{1 - Y^2} \quad (3.18)$$

$$P = 1 - X \quad (3.19)$$

The solution of 3.12 can be written

$$n^2 = \frac{B + mF}{2A} \quad F = \sqrt{B^2 - 4AC} \quad (3.20)$$

and $m = \pm 1$ denotes the propagation mode. As $n_e \rightarrow 0$, $B \rightarrow 2$, $A \rightarrow 1$, and $F \rightarrow 0$ whereupon n takes the value of unity for free space. Thus, the condition $B^2 = 4AC$ is the plasma/vacuum ‘crossing point’ and this is checked in the code to determine where to transfer from vacuum to plasma dispersion relations (or vice-versa).

inside plasma $|B^2 - 4AC| > EPS$

outside plasma $|B^2 - 4AC| \leq EPS$

where EPS is a very small number.

Once inside the plasma, the code must also check if one or both of the modes from the plasma dispersion relation 2.11 is cutoff. In general, the condition for a propagating mode to become cutoff in the absence of any damping mechanisms is $n^2 \rightarrow 0$. Thus, from equation 3.20 the condition

$$B + mF \stackrel{?}{<} 0 \quad (3.21)$$

is checked for each mode ($m = \pm 1$) to determine if it will continue to be followed in the calculation or reported as cutoff.

Application

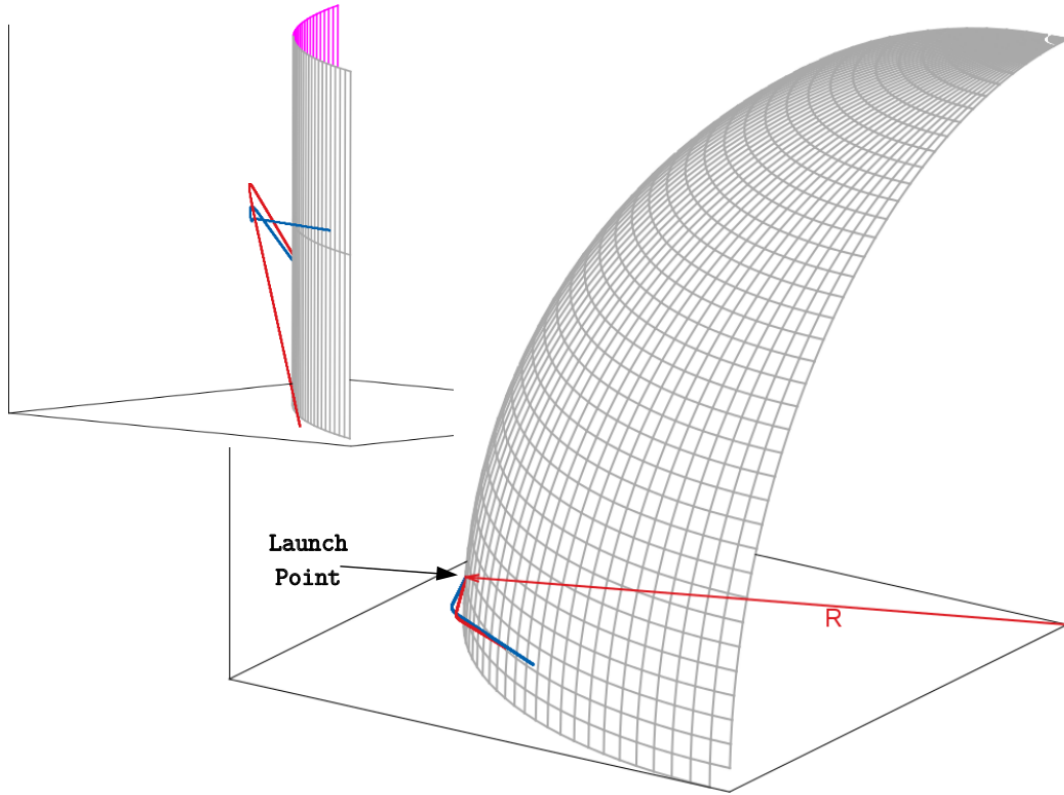


Figure 4.1: Ray tracing a 39.7 MHz $m = +1$ ray from the Earth's surface.

To exercise the code, we start with the well known Chapman electron density profile [3]

$$n_e = n_0 \cdot \exp \left[\frac{1}{2} \cdot \left(1.0 - \frac{R - R_{max}}{h} - e^{-(R - R_{max})/h} \right) \right] \quad (4.1)$$

where the radius vector in the ECEF system is

$$R = [X^2 + Y^2 + Z^2]^{1/2} \quad (4.2)$$

n_0 is the local electron density, R_{max} is the height of the maximum in electron density set to $R_E + 300$ km, R_E is the Earth's radius, and h is a 'thickness' parameter set to 35 km. To that,

we add a density depletion centered at some X_0, Y_0, Z_0 point above the Earth's surface such that if the ray passes within a distance σ , the local electron density falls to

$$n_l = n_e - \delta n \cdot \exp \left[- \left\{ (X - X_0)^2 + (Y - Y_0)^2 + (Z - Z_0)^2 \right\} / \sigma^2 \right] \quad (4.3)$$

This is basically a Gaussian density depletion bubble centered at X_0, Y_0, Z_0 .

The Earth's geometry is assumed spherical. The Earth's magnetic field is a simple dipole field centered on the sphere's north and south poles. Note that the code can accept any geometry for the Earth and geomagnetic field, the particular choices for the examples in this report were chosen for convenience.

Figure 4.1 shows the results of launching a $m = +1$ mode ray of frequency 39.7 MHz at a location of $\theta, \phi = 85^\circ, 35^\circ$ on the Earth's surface for two oblique launch angles. The maximum electron density was $n_0 = 3.0 \cdot 10^{12} m^{-3}$ with $\delta n = -3.0 \cdot 10^{12} m^{-3}$, an aggressive density depletion within the bubble. Both rays are refracted back to Earth, but take different paths.

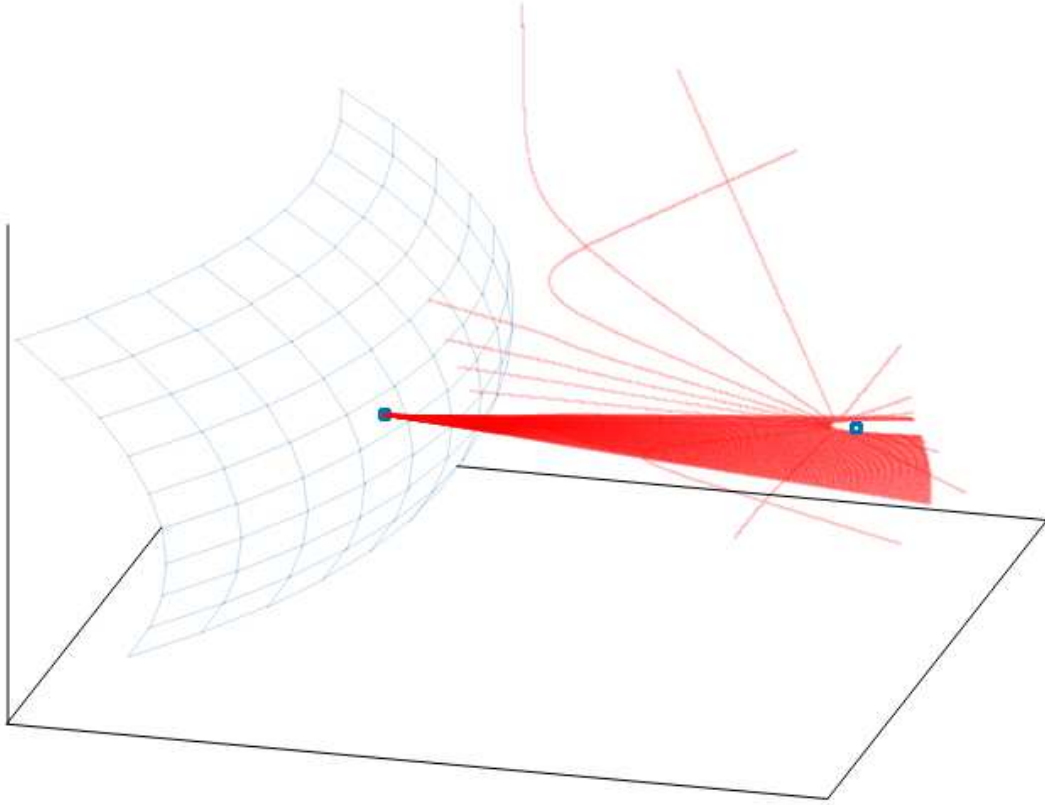


Figure 4.2: Ray tracing 10 MHz $m = +1$ rays from the Earth's surface.

Figure 4.2 shows the results of launching several $m = +1, 10$ MHz rays to a similar Gaussian density depletion ($\sigma = 20$ km) located 1000 km above Earth at $\theta, \phi = 85^\circ, 85^\circ$. The launch angles are all in the $\theta = 85^\circ$ plane in the ECEF system. These distances, and the Gaussian symmetry in the ionosphere and density depletion, dictate that the ray refraction is mostly in the $\theta = 85^\circ$ plane.

In figure 4.3, the code was run only in two dimensions with a local depletion, enhancement, and no perturbation in the electron density. Here we see that a detector traveling at an altitude

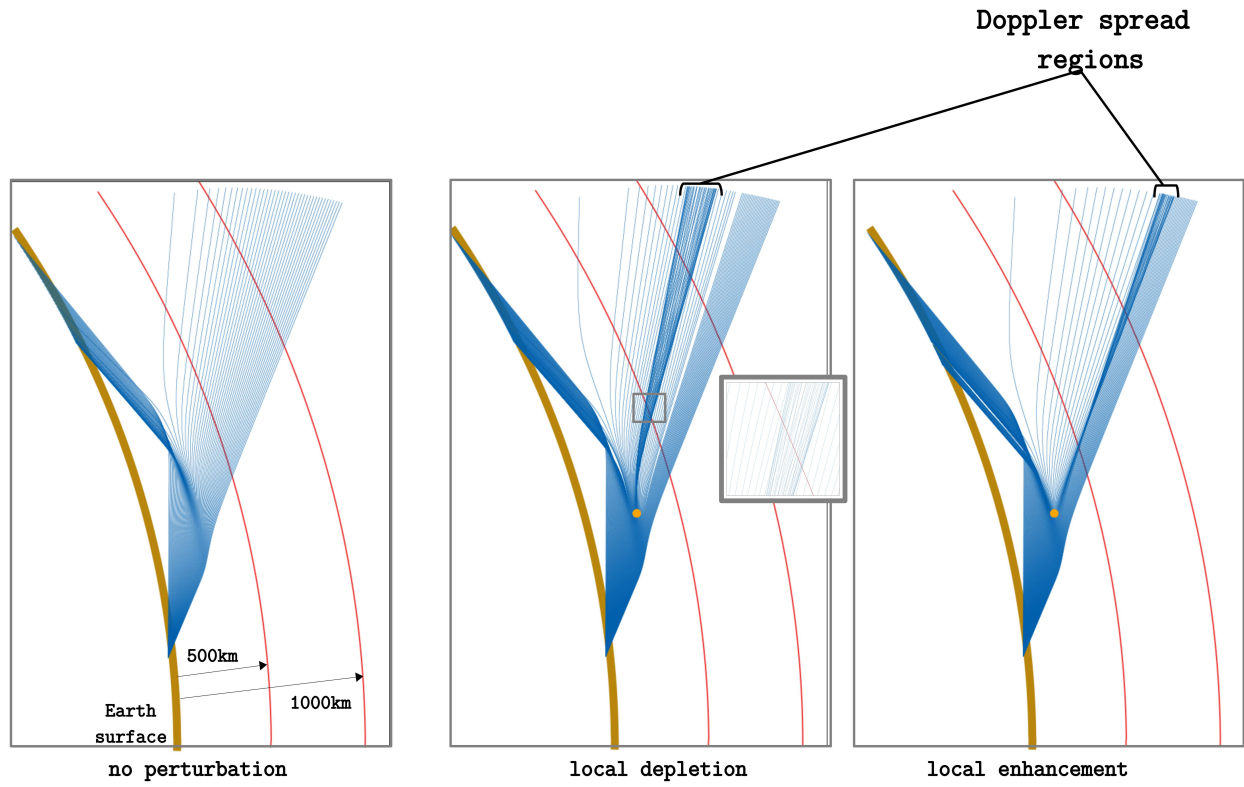


Figure 4.3: Ray tracing 39.7 MHz $m = +1$ rays from the Earth's surface - doppler spread effects

above about approx. 500 km in this example will experience the doppler spread effect. That is, the single frequency ray will reach the detector at different times due to their paths crossing before reaching the detector, causing a difference in the doppler shift from the signal.

Summary

We have developed a fully three dimensional ray tracing code for arbitrary electron density and geomagnetic field profiles. The algorithm is based on one used previously [8]. The code has been exercised and gives reasonable results based on the chosen ionospheric parameters for the examples presented.

Bibliography

- [1] D. Swanson, *Plasma Waves, 2nd Edition*. Series in Plasma Physics, Taylor & Francis, 2003.
- [2] T. Stix, *Waves in Plasmas*. American Inst. of Physics, 1992.
- [3] K. G. Budden, *Radio Waves in the Ionosphere*. Cambridge University Press, 1985.
- [4] K. Yeh and C. Liu, *Theory of Ionospheric Waves*. International Geophysics, Elsevier Science, 1973.
- [5] I. B. Bernstein, “Geometric optics in time and space varying plasmas,” *Physics of Fluids*, vol. 18, p. 320, 1975.
- [6] S. Weinberg, “Eikonal method in magnetohydrodynamics,” *Physical Review*, vol. 126, p. 1899, 1962.
- [7] Y. A. Kravtsov, *Geometrical Optics in Engineering Physics*. Alpha Science International Ltd., 2005.
- [8] R. B. Horne, “Path-integrated growth of electrostatic waves: The generation of terrestrial myriametric radiation,” *Journal of Geophysical Research*, vol. 94, no. A7, pp. 8895 – 8909, 1989.